Engineering Serendipity: Successes in Solitary Foraging and New Investigations in Cooperative Task-Processing Networks

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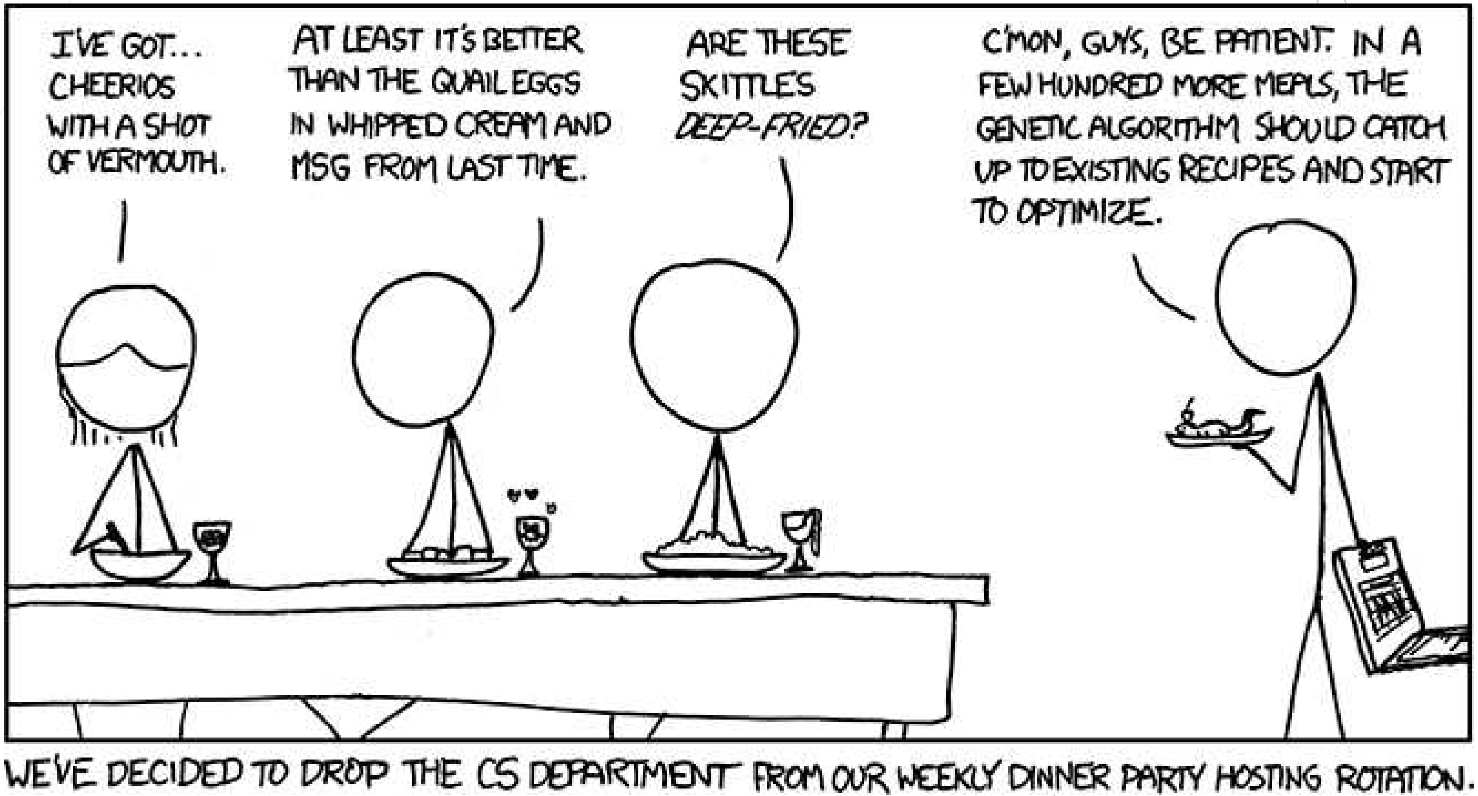
Introduction
- Reflections
- Faulty connections
- Missed connections
- Motivations

Solitary foraging: from ecology to engineering and back
- Speed choice ($\rightarrow$)
- Impulsiveness and operant conditioning ($\leftarrow$)
- Long patch residence times ($\leftarrow$)

Cooperative task processing
- Background
- Task-processing network
- Cooperation game
- Asynchronous convergence to cooperation
- Results

Closing remarks

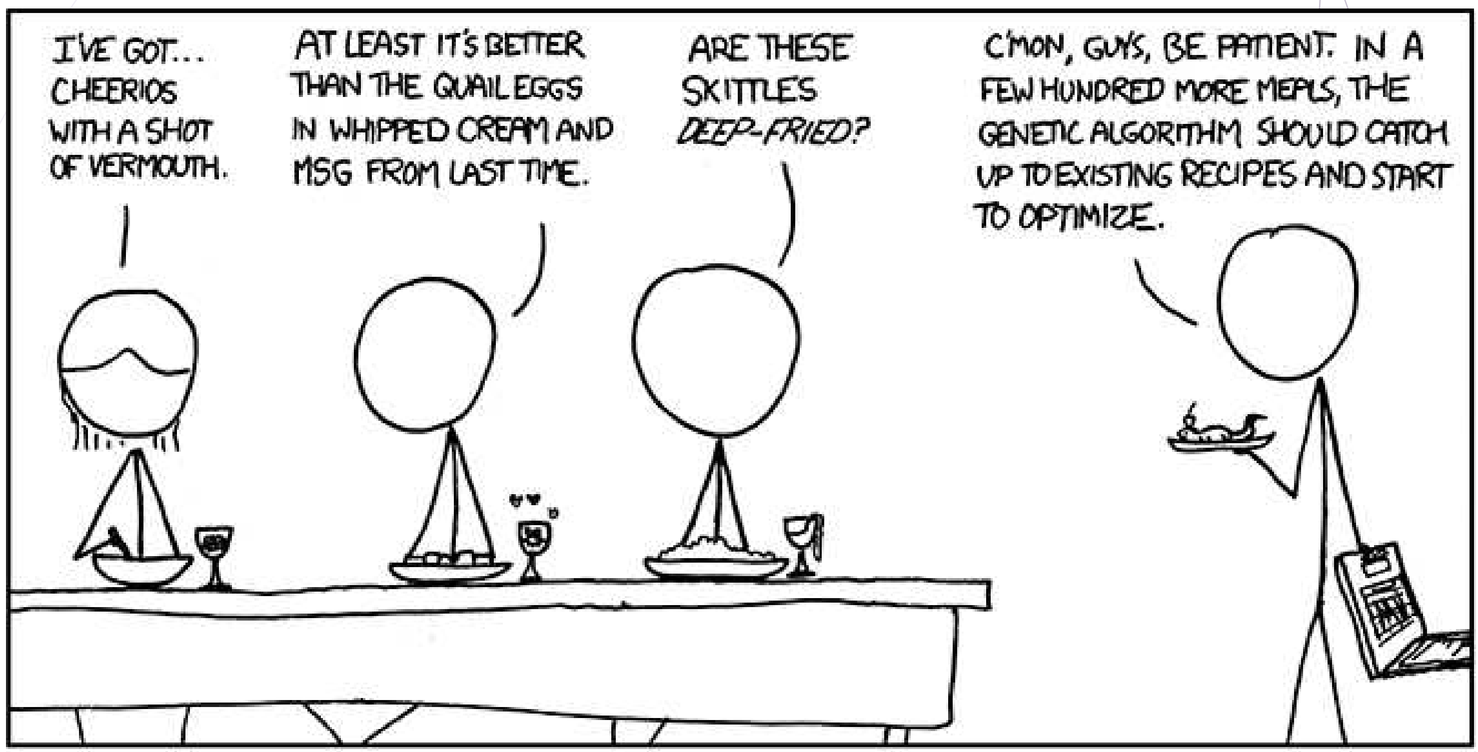


[1]

- Craig Tovey: “manufacturing serendipity”

[1] Compliments to XKCD: http://xkcd.com/720/.

Engineering~~Manufacturing~~ Serendipity



WE'VE DECIDED TO DROP THE CS DEPARTMENT FROM OUR WEEKLY DINNER PARTY HOSTING ROTATION.[1]

- Craig Tovey: success with bees and Internet server allocation

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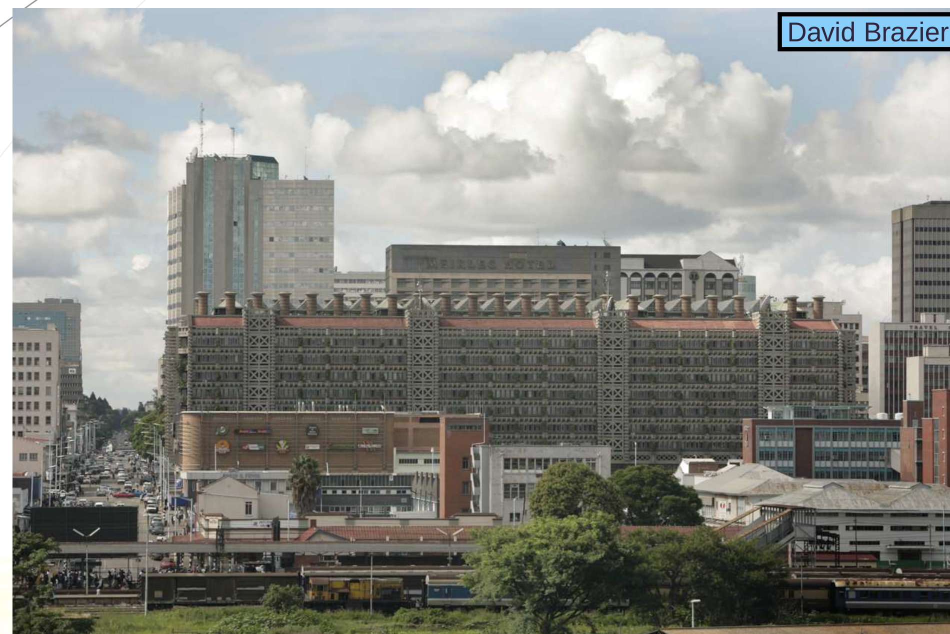
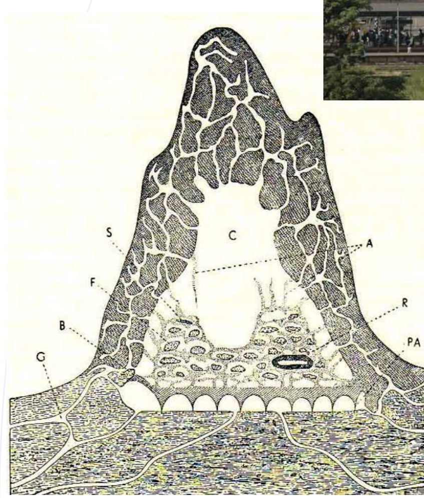
Faulty connections





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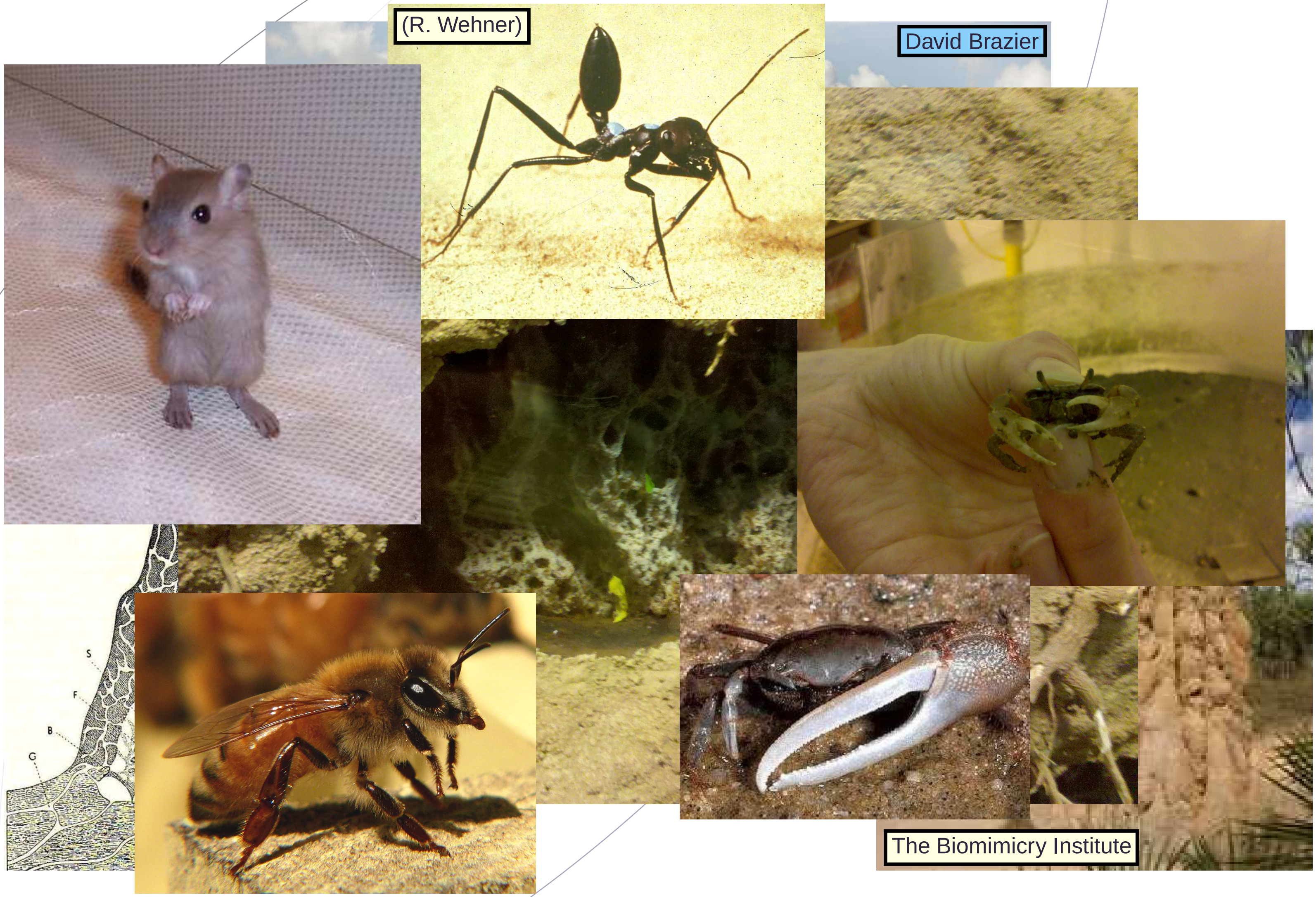

Missed Abstract Connections

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$$\text{minimize} \quad \sum_{i=1}^{n} C_i(P_i) \qquad \text{subject to} \quad \sum_{i=1}^{n} P_i = P$$

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  Solution (from KKT) is an "upside-down" IFD:

  $$\frac{\mathrm{d}C_i(P_i)}{\mathrm{d}P_i} = \lambda \quad \forall i \in \{1, 2, \ldots, n\} \quad \text{(and truncate appropriately)}$$

  Equalization of marginal cost matches IFD equalization of suitability.

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  IFD as optimization problem (thought experiment):

  $$\text{maximize} \quad \sum_{i=1}^{n} \int_0^{x_i} s_i(y)\,\mathrm{d}y \quad \text{subject to} \quad \sum_{i=1}^{n} x_i = N$$

  Pareto maximization of (???) subject to conservation simplex.

  Right-side-up IFD:

  $$s_i(x_i) = \lambda \quad \forall i \in \{1, 2, \dots, n\} \quad \text{(and truncate appropriately)}$$

  Distribute x_i to equalize suitability.

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  IFD as optimization problem (thought experiment):

  $$\text{maximize} \quad \sum_{i=1}^{n} G_i(x_i) \qquad \text{subject to} \quad \sum_{i=1}^{n} x_i = N$$

  Pareto maximization of gain(?) subject to conservation simplex.

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  $$\frac{\mathrm{d}G_i(x_i)}{\mathrm{d}x_i} = \lambda \quad \forall i \in \{1, 2, \dots, n\} \quad \text{(and truncate appropriately)}$$

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  IFD as optimization problem (thought experiment):

  $$\text{maximize} \quad \sum_{i=1}^{n} G_i(t_i) \qquad \text{subject to} \quad \sum_{i=1}^{n} t_i = T$$

  Maximization of distributed gain subject to limited time inside patch.

  Right-side-up IFD:

  $$\frac{\mathrm{d}G_i(t_i)}{\mathrm{d}t_i} = \lambda \quad \forall i \in \{1, 2, \ldots, n\} \quad \text{(and truncate appropriately)}$$

  Distribute t_i to equalize marginal gain.

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- IFD (Fretwell 1972; Fretwell and Lucas 1969) $\iff$ Optimal power dispatch (Bergen and Vittal 2000)
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  MPT (then) $\rightarrow$ PMPT (now) (stochastic dominance, Bawa 1982)

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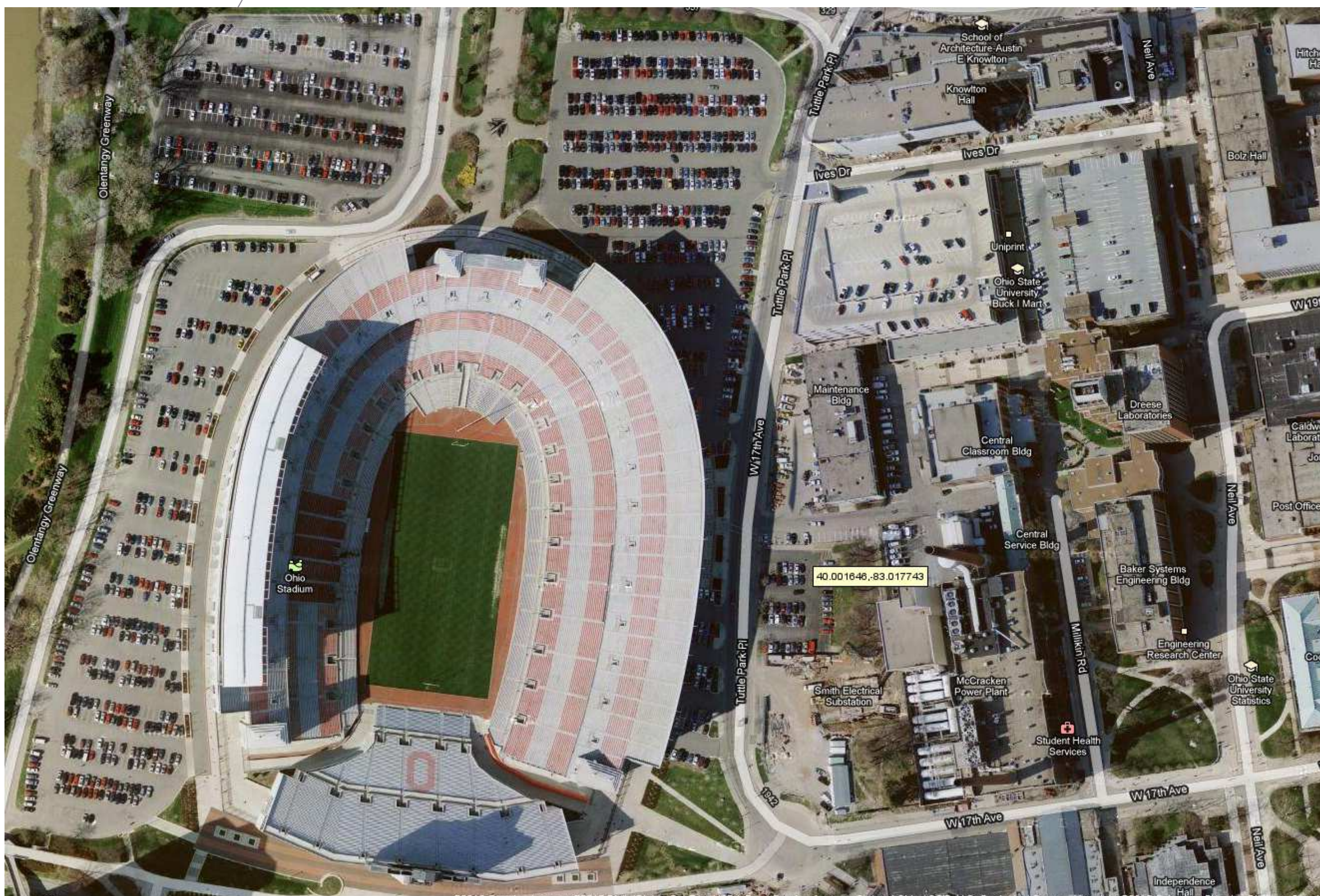
- Iain Couzin's (2000$+$) $\Longleftrightarrow$ Bertsekas and Tsitsiklis (1997$-$) (e.g., mysterious torus shapes; symmetry; symmetry breaking)

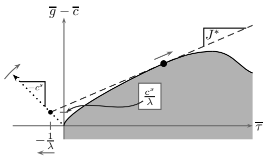
The Tenth Muse



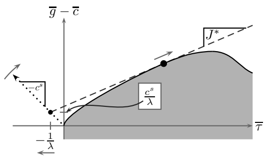
Solitary foraging: from ecology to engineering and back

Foraging theory for speed choice

- Nice homomorphism between solitary foragers and autonomous vehicles (Andrews *et al.* 2004; Charnov 1973; Quijano *et al.* 2006; Stephens and Krebs 1986)



Foraging theory for speed choice



- Nice homomorphism between solitary foragers and autonomous vehicles (Andrews *et al.* 2004; Charnov 1973; Quijano *et al.* 2006; Stephens and Krebs 1986)
  - Fitness surrogate (e.g., calories, target value)
  - Diverse collection of targets
    - Opportunity cost: some should be ignored
  - Rate maximization for long runs
  - Target/task choice $\iff$ prey model

Foraging theory for speed choice

- Nice homomorphism between solitary foragers and autonomous vehicles (Andrews *et al.* 2004; Charnov 1973; Quijano *et al.* 2006; Stephens and Krebs 1986)
- Vehicle speed choice is very similar to cryptic prey problem described by Gendron and Staddon (1983)
  - *Ceteris paribus*, encounter rate increases with search speed
  - Search cost increases with search speed
  - Detection mistakes may vary with speed
  - Non-trivial speed–prey choice coupling
    - Prey $\Longrightarrow$ speed $\Longrightarrow$ rate $\Longrightarrow$ prey

Bobwhite quail (Gendron and Staddon 1983)

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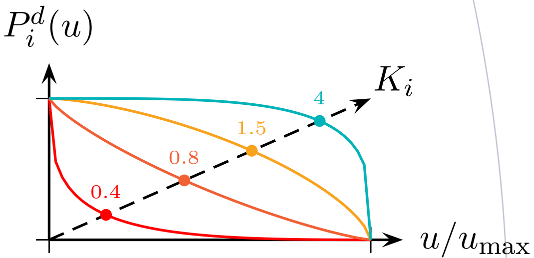
Bobwhite quail (Gendron and Staddon 1983)

- Vehicle speed choice is very similar to cryptic prey problem described by Gendron and Staddon (1983)
- To match bobwhite quail observations, Gendron and Staddon choose detection function $P_i^d(u) \triangleq (1 - (u/u_{\text{max}})^{K_i})^{1/K_i}$ that maps search speed $u \in [0, u_{\text{max}}]$ to detection probability P_i^d for tasks of type i with conspicuousness $K_i \in [0, \infty)$.
  - No analytical tractability
  - Chose $n = 2$ for simulation (1983)
  - P_i^d is strange at bounds (1 and 0)



On-line prey–speed choice for $n \in \mathbb{N}$ (Pavlic 2007; Pavlic and Passino 2009)

- Autonomous vehicle faces n-way merged Poisson process
  - λ_i: encounter rate for task of type i
  - (g_i, t_i): average (value, time) for processing task of type i
  - p_i: probability that task of type i is processed (decision)
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- Vehicle goes through cycles of searching and processing
  - $\bar{G}$: average per-encounter gain
  - $\bar{T}$: average per-encounter search and processing time
  - $\mathcal{G}(t)$: Markov renewal–reward process for accumulated gain

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- Long runtime $\implies$ maximize rate of return

$$\operatorname*{aslim}_{t \to \infty} \frac{\mathcal{G}(t)}{t} = \frac{\bar{G}}{\bar{T}} = \frac{-c^s + \sum_{i=1}^{n} \lambda_i p_i g_i}{1 + \sum_{i=1}^{n} \lambda_i p_i t_i} \triangleq R(\mathbf{p})$$

As expected, **type-II** functional response (Holling's disk equation without any sandpaper disks).

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- In general, $p_i \in [0, 1]$, but

$$\frac{\partial R(\mathbf{p})}{\partial p_i} = \frac{\lambda_i g_i \left(1 + \sum_{j=1}^{n} \lambda_j p_j t_j\right) - \lambda_i t_i \left(-c^s + \sum_{j=1}^{n} \lambda_j p_j g_j\right)}{\left(1 + \sum_{i=1}^{n} \lambda_i p_i t_i\right)^2}$$

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So-called *zero–one rule* because $p_i^* \in \{0, 1\}$

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[Sufficient condition; not necessary]

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- Classical prey ranking refines search from $O(2^n)$ to $O(n+1)$

$$\underbrace{\frac{g_1}{t_1} > \frac{g_2}{t_2} > \dots > \frac{g_{k^*}}{t_{k^*}}}_{\text{Processed types } (p_i^* = 1)} > \overbrace{\frac{-c^s + \sum_{i=1}^{k^*} \lambda_i g_i}{1 + \sum_{i=1}^{k^*} \lambda_i t_i}}^{\text{Optimal rate } R(\mathbf{p}^*)} > \underbrace{\frac{g_{k^*+1}}{t_{k^*+1}} > \dots > \frac{g_n}{t_n}}_{\text{Ignored types } (p_i^* = 0)}$$

where optimal $p_i^* = [i \leq k^*]$ with $k^* \in \{0, 1, \dots, n\}$

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- Classical prey ranking **does not depend on λ (i.e., speed)**

$$\underbrace{\frac{g_1}{t_1} > \frac{g_2}{t_2} > \ldots > \frac{g_{k^*}}{t_{k^*}}}_{\text{Processed types } (p_i^* = 1)} > \underbrace{\frac{-c^s + \sum_{i=1}^{k^*} \lambda_i g_i}{1 + \sum_{i=1}^{k^*} \lambda_i t_i}}_{\text{Optimal rate } R(\mathbf{p}^*)} > \underbrace{\frac{g_{k^*+1}}{t_{k^*+1}} > \ldots > \frac{g_n}{t_n}}_{\text{Ignored types } (p_i^* = 0)}$$

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- Speed $u \in [u_{\text{min}}, u_{\text{max}}] \subset [0, \infty)$ influences each encounter rate

$$\lambda_i(u) = u D_i P_i^d(u)$$

where D_i is the linear density in the population

Effects of speed

- Speed $u \in [u_{\text{min}}, u_{\text{max}}] \subset [0, \infty)$ influences each encounter rate

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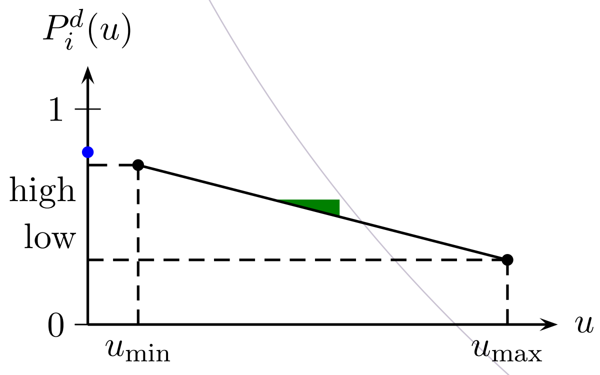
- Detection function is linear interpolation of probability bounds



$$P_i^d(u) = P_i^\ell u + P_i^a$$

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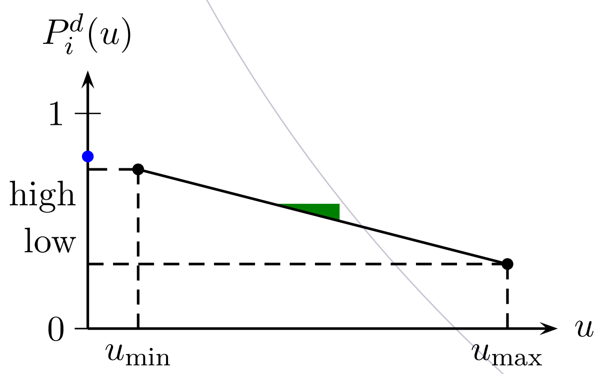

$$P_i^d(u) = P_i^\ell u + P_i^a$$

- Search cost is also assumed to be affine function

$$c^s(u) = c_\ell^s u + c_a^s$$

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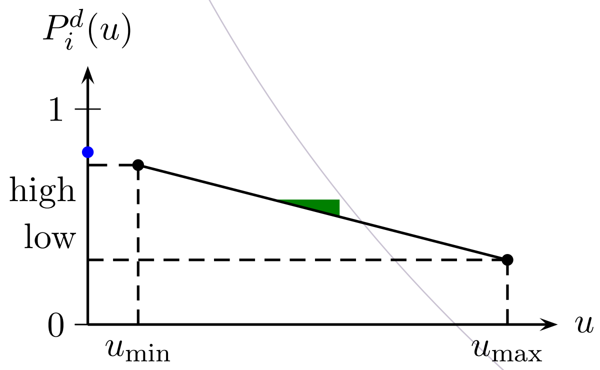

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- *[Processing costs can be modeled in a similar way]*

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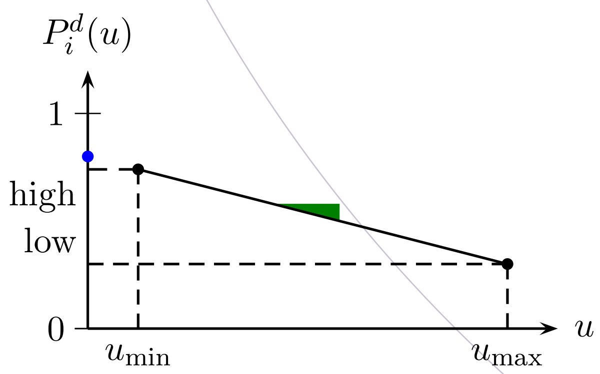

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- *[Processing costs can be modeled in a similar way]*

$$c^s(u) = c_\ell^s u + c_a^s$$

[. . . but not here.]

- After regrouping, new objective function

$$R(\mathbf{p}, u) = \frac{G_2(\mathbf{p})u^2 + G_1(\mathbf{p})u + G_0(\mathbf{q})}{T_2(\mathbf{p})u^2 + T_1(\mathbf{p})u + 1}$$

where coefficients

$$G_2(\mathbf{p}) \triangleq \sum_{i=1}^{n} D_i p_i g_i P_i^{\ell} \qquad T_2(\mathbf{p}) \triangleq \sum_{i=1}^{n} p_i t_i D_i P_i^{\ell}$$

$$G_1(\mathbf{p}) \triangleq \sum_{i=1}^{n} D_i p_i P_i^{a} g_i - c_{\ell}^{s} \qquad T_1(\mathbf{p}) \triangleq \sum_{i=1}^{n} p_i t_i D_i P_i^{a}$$

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are constant with respect to u (i.e., biquadratic ratio)

- Find optimal u^* for each $\mathbf{p}^*$ candidate ($n+1$ total)

Finding optimal speed

- Because biquadratic objective, for each $\mathbf{p}^*$ candidate,

$$\frac{\partial R(u)}{\partial u} = \frac{(G_2 T_1 - G_1 T_2) u^2 + 2(G_2 - G_0 T_2) u + (G_1 - G_0 T_1)}{\left(T_2 u^2 + T_1 u + 1 \right)^2}$$

By KKT, if quadratic numerator root $u^* \in [u_{\text{min}}, u_{\text{max}}]$, then u^* is optimal speed; otherwise, optimal speed $u^* \in \{u_{\text{min}}, u_{\text{max}}\}$ based on sign of numerator

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- Implement $O(n+1)$ algorithm on-line if D_i density estimates available (Dubin's car AAV simulations with speed filtering, Pavlic and Passino 2009)

- Non-trivial to guarantee convergence of density estimates on-line
  - Estimation process $\Longrightarrow$ ~~type-II~~ **type-III** functional response

Estimation, impulsiveness, and the operant laboratory (Pavlic and Passino 2010c)

- Laboratory impulsiveness (Ainslie 1974; Bateson and Kacelnik 1996; Bradshaw and Szabadi 1992; Green *et al.* 1981; McDiarmid and Rilling 1965; Rachlin and Green 1972; Siegel and Rachlin 1995; Snyderman 1983; Stephens and Anderson 2001)

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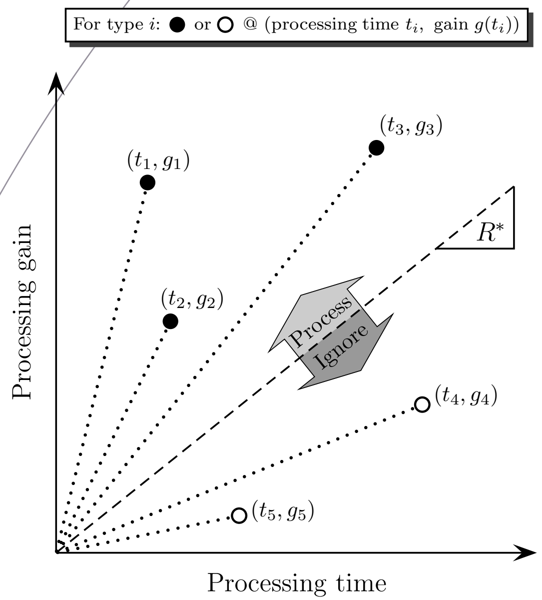

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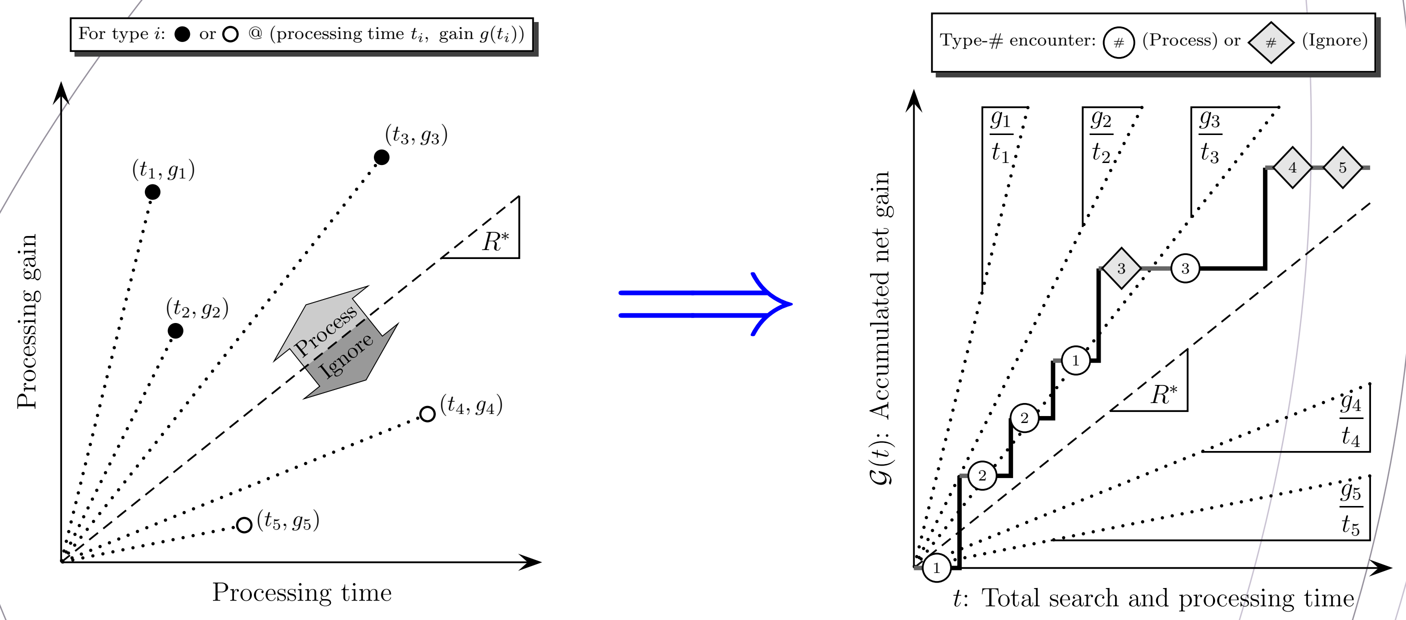

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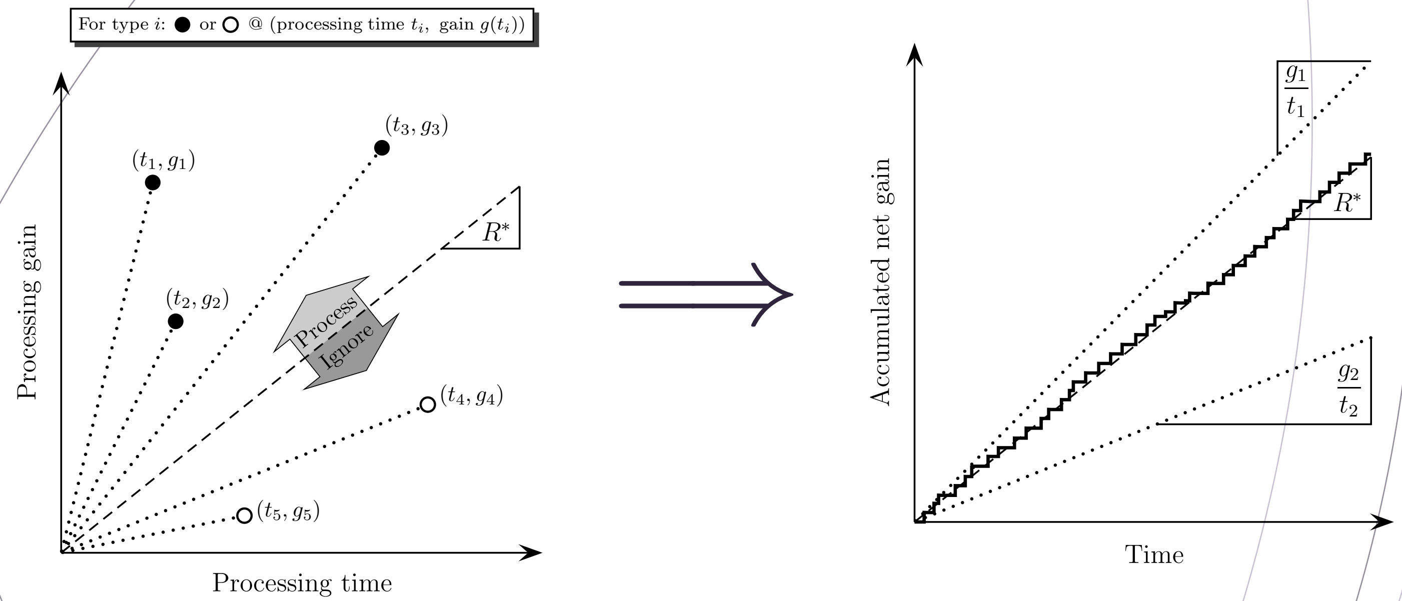

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- Rule (even with mistakes) is optimal facing Poisson encounters (i.e., simultaneous w.p.0)

- Estimation of per-type densities only necessary for speed regulation

- Digestive rate constraints (b_i: prey bulk) (Hirakawa 1995):

$$\frac{\sum_{i=1}^{n} \lambda_i p_i b_i}{1 + \sum_{i=1}^{n} \lambda_i p_i t_i} \leq B \quad \overset{\text{KKT}}{\Longrightarrow} \quad \begin{aligned} p_1^* &= 1 \\ &\vdots \\ p_{k^*-1}^* &= 1 \\ p_{k^*}^* &\in [0, 1] \end{aligned}$$

Partial Preferences (rank by g_i/b_i)

Digression

- Estimation of per-type densities only necessary for speed regulation

- Ecological–physiological hybrid method (Whelan and Brown 2005):

$$\text{Asymptotic gut constraint} \iff \text{Rank by } \frac{g_i}{t_i + t_i^b}$$

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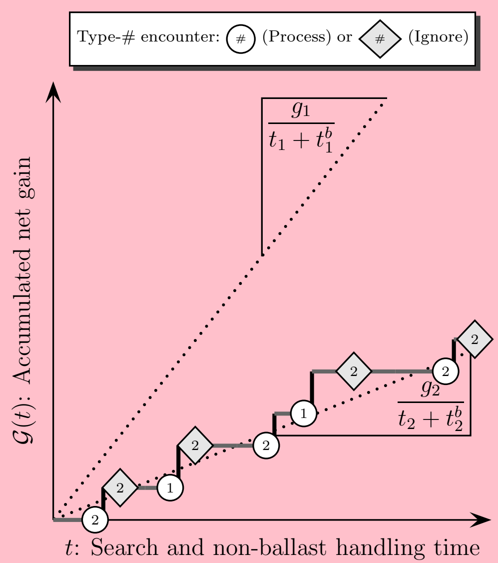

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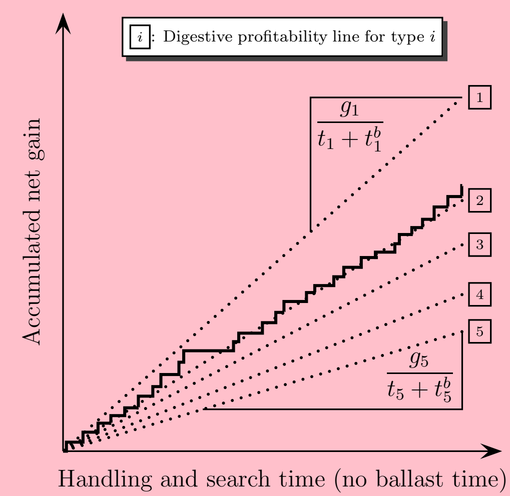

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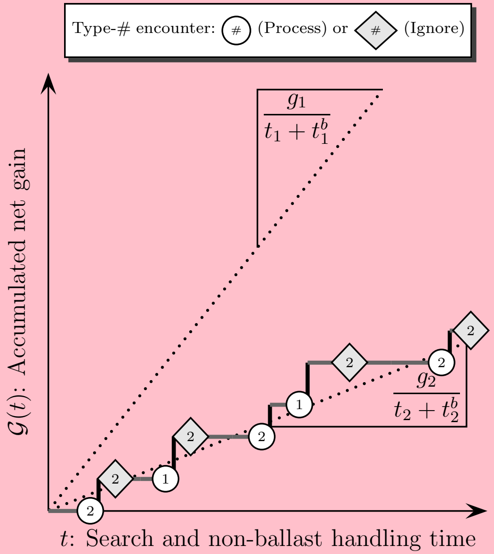

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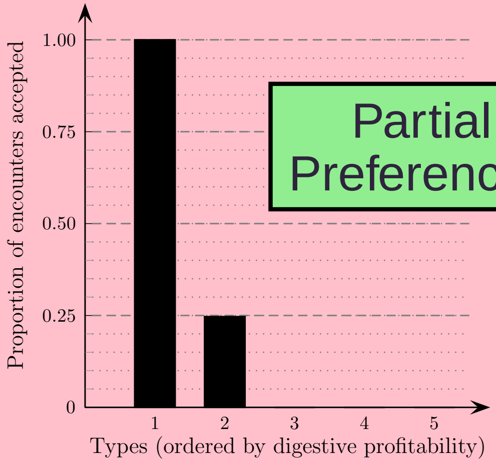

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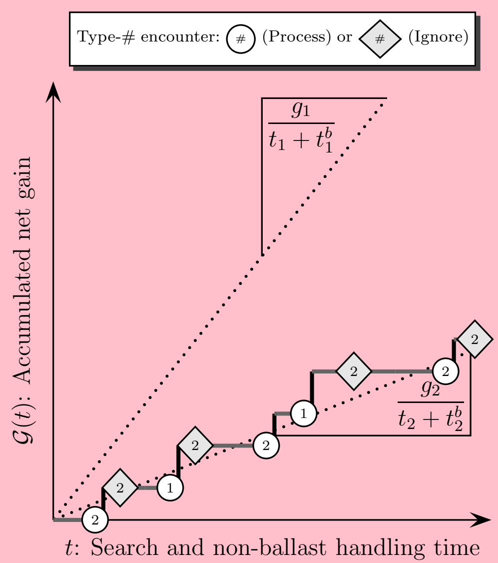

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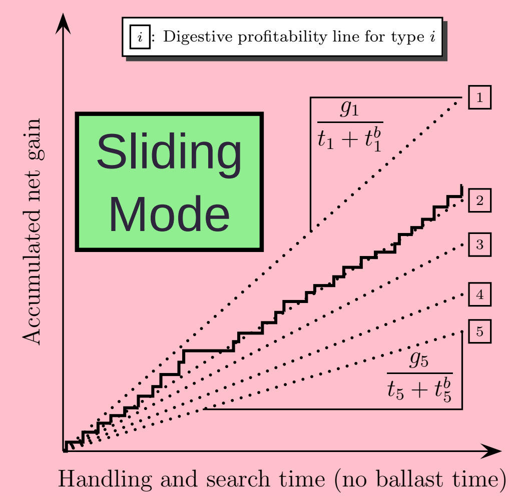

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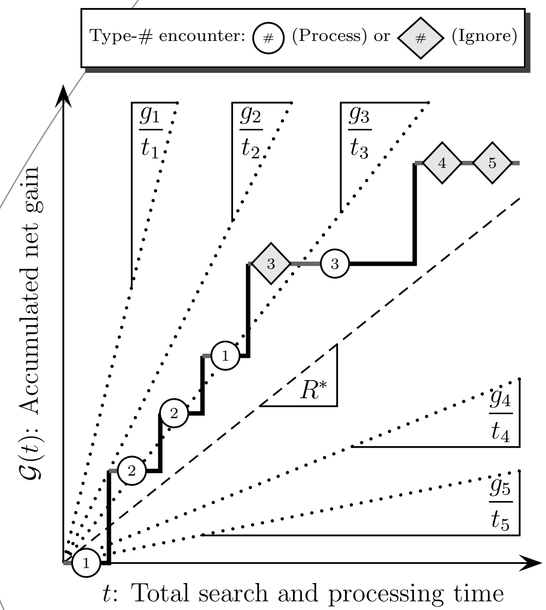

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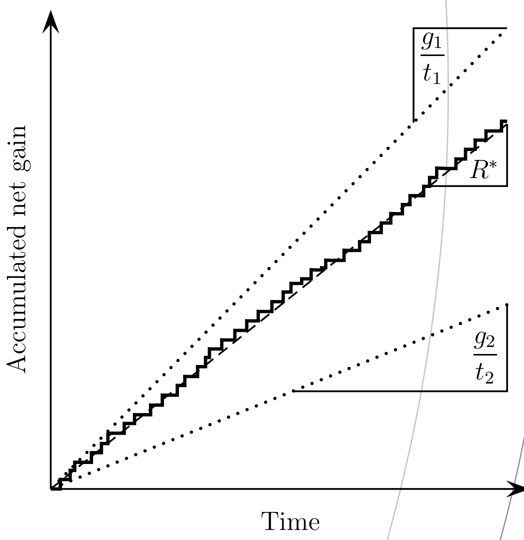

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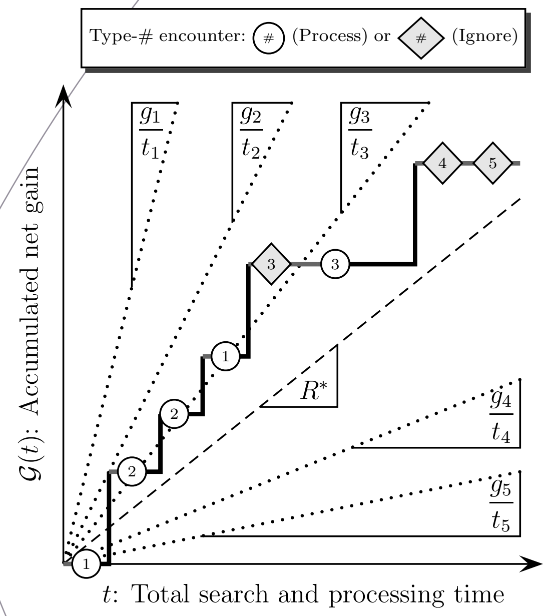

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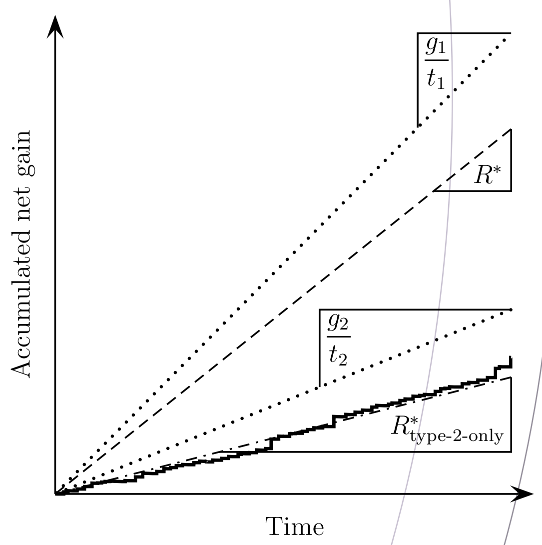

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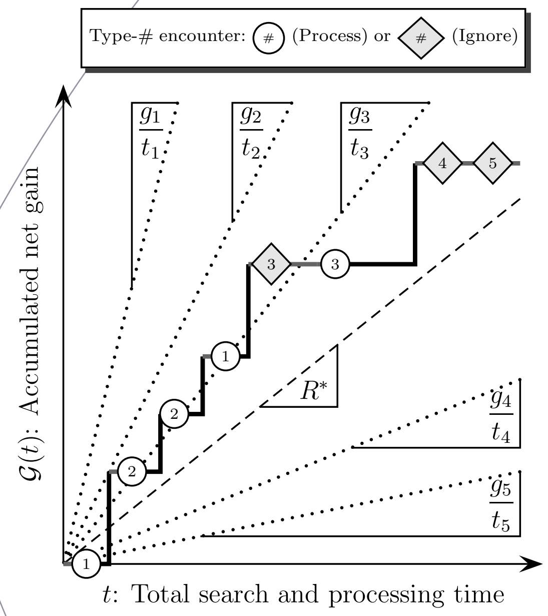

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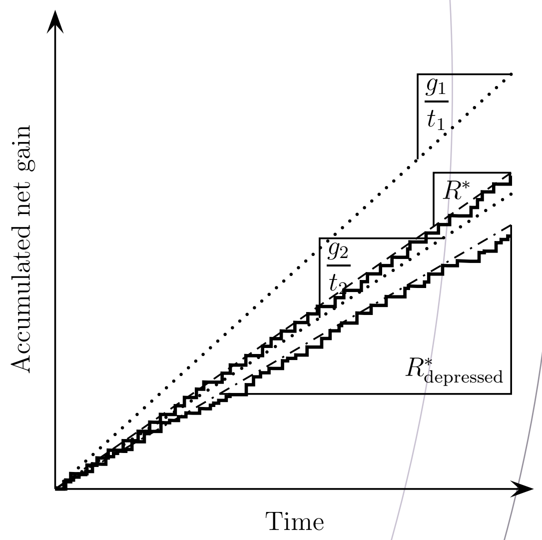

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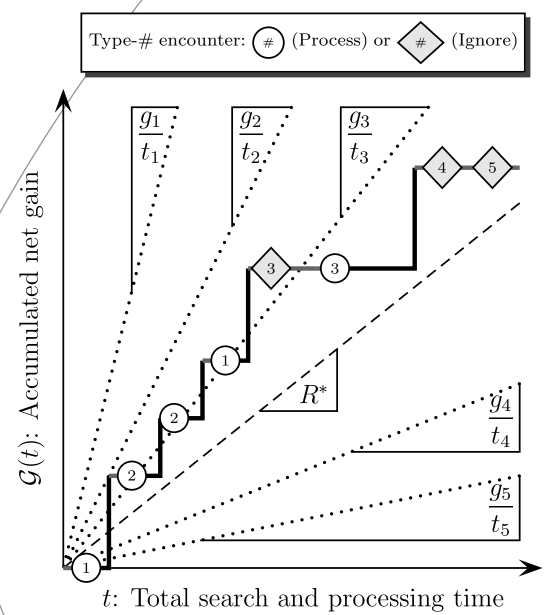

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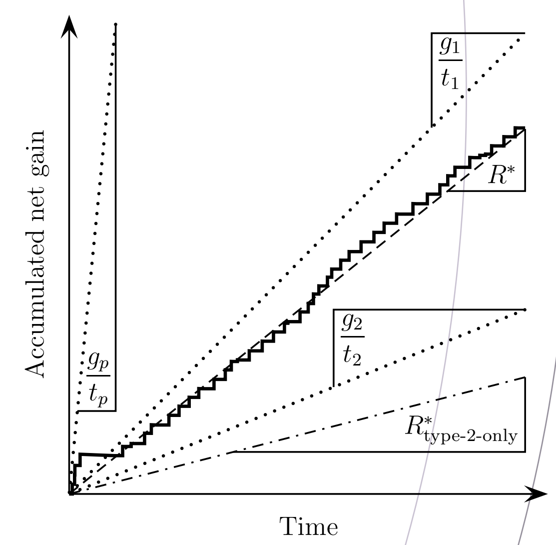

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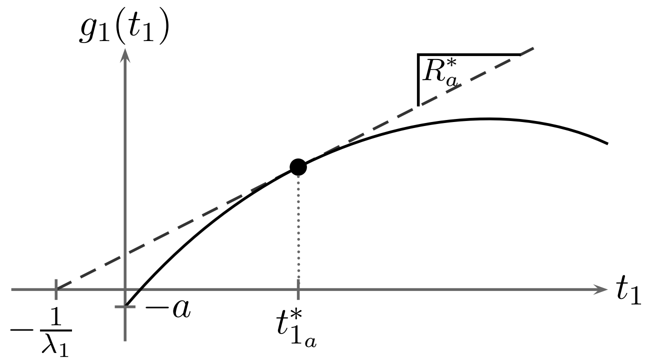

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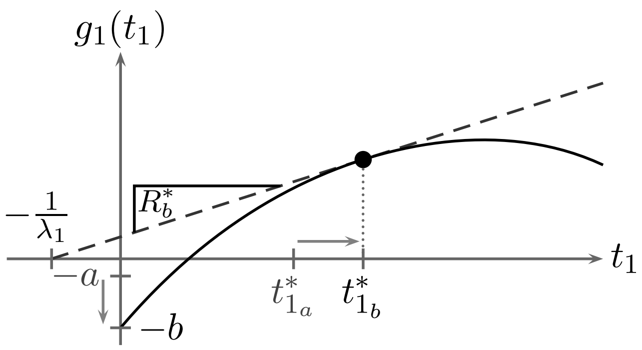

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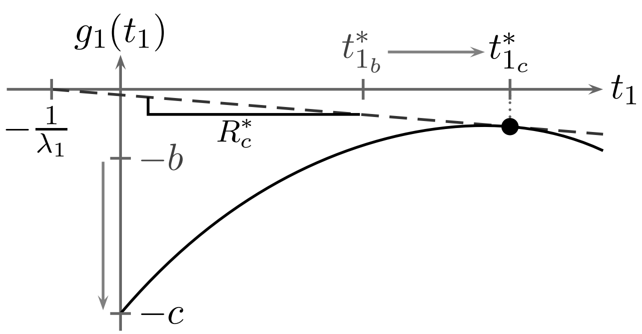

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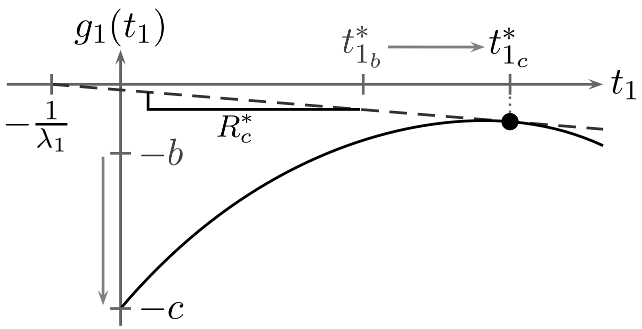

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- May explain overstaying as well (Nonacs 2001)

Cooperative task processing

Cooperation for distributed decentralized networks

- Cooperative control usually involves coordination of agents on (possibly *ad hoc*) networks
  - e.g., Global utility functions to maximize
  - e.g., Projections onto non-separable spaces (i.e., not Cartesian products)
  - Challenges to fast and cheap implementation

Cooperation for distributed decentralized networks

- Cooperative control usually involves coordination of agents on (possibly *ad hoc*) networks
- Nash (i.e., competitive) equilibria are solutions to separable variational inequality problems
  - Amenable to parallel solvers
  - Used by communication theorists on networks for congestion control (Altman *et al.* 2005a,b; Buttyán and Hubaux 2003; Shakkottai *et al.* 2006)
  - Strong connection to biological (and sociological) models of emergent cooperation in nature

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- Used in control to model unknown/unknowable
  - Typically used in control to model noise or enemy movements (e.g., worst-case scenarios) or actions of humans in the system
  - Task conservation is a challenge to communication-like application of Nash methods to task flow control

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- Existing task-processing networks (TPN) (Cruz 1991; Perkins and Kumar 1989) focus on robustness, not optimality:
  - Flexible manufacturing system, network components $\Longrightarrow$ bounded queues/burstiness
  - Behaviors are static (i.e., no feedback)

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- So here, elements merged from communication, TPN, and possible analogous systems in nature (e.g., Cooperative breeding, Hamilton and Taborsky 2005)
  - Try to design system so that Nash equilibrium has characteristics that are globally favorable

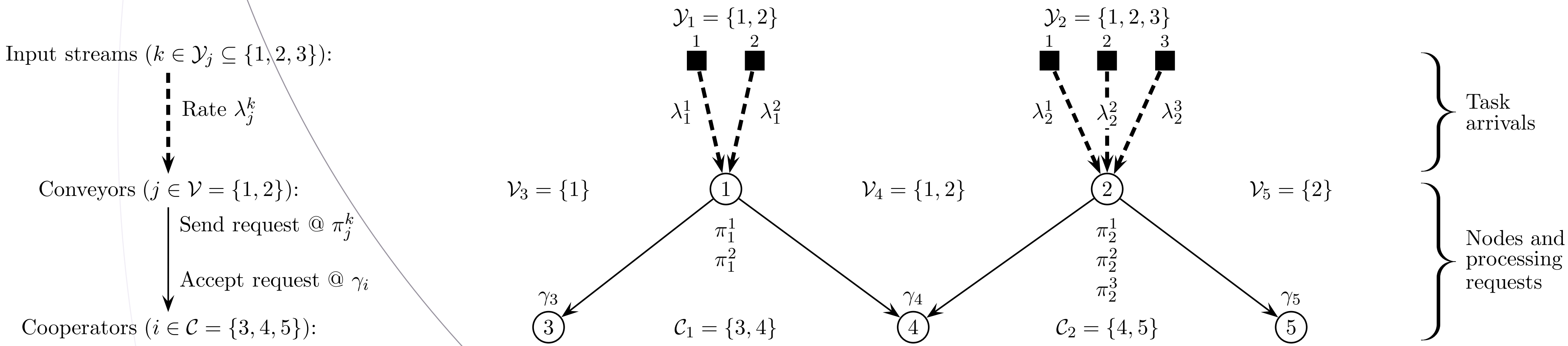
A task-processing network is a directed graph:

- $\mathcal{A} \subset \mathbb{N}$: Set of *task-processing agents*
- $\mathcal{P} \subseteq \{(i,j) \in \mathcal{A}^2 : i \neq j\}$: Directed arcs connecting distinct agents
- $\mathcal{V}_i \triangleq \{j \in \mathcal{A} : (j,i) \in \mathcal{P}\}$: Set of *conveyors* for each $i \in \mathcal{A}$
- $\mathcal{C}_i \triangleq \{j \in \mathcal{A} : (i,j) \in \mathcal{P}\}$: Set of *cooperators* for each $i \in \mathcal{A}$
- $\mathcal{V} \triangleq \{j \in \mathcal{A} : \mathcal{C}_j \neq \emptyset\}$: Set of all conveyors
- $\mathcal{C} \triangleq \{i \in \mathcal{A} : \mathcal{V}_i \neq \emptyset\}$: Set of all cooperators

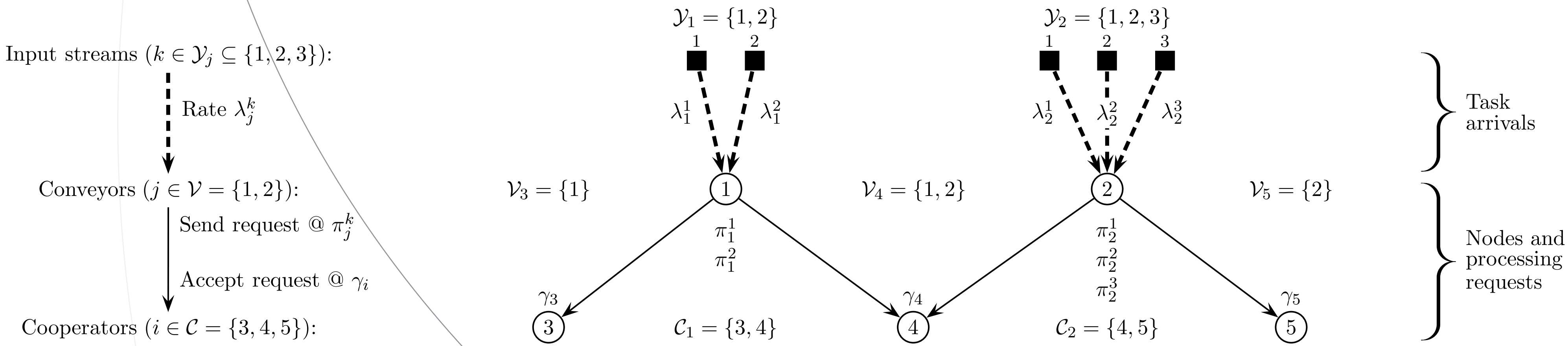
Task flows at each agent:

- $\mathcal{Y}_i \subset \mathbb{N}$: Possibly empty set of *task types* that arrive at conveyor $i \in \mathcal{A}$
- $\lambda_j^k \in \mathbb{R}_{>0}$: Encounter rate of type-k tasks at agent $j \in \mathcal{A}$ (e.g., Poisson encounters)
- $\pi_j^k \in [0,1]$: Probability that conveyor $j \in \mathcal{A}$ advertises an incoming k-type task to its connected cooperators $\mathcal{C}_j$
- $\gamma_i \in [0,1]$: Probability that cooperator $i \in \mathcal{A}$ volunteers for advertised task from one of its connected conveyors $\mathcal{V}_i$ (collected in γ)

TPN examples (Pavlic and Passino 2010a)



Flexible manufacturing system (FMS)



~~Flexible manufacturing system (FMS)~~

Cooperative breeders?

TPN examples
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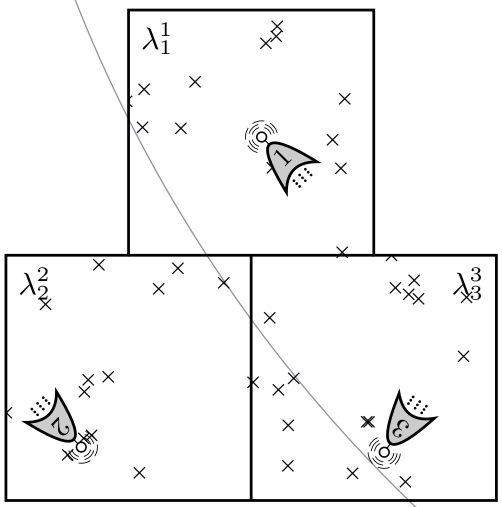
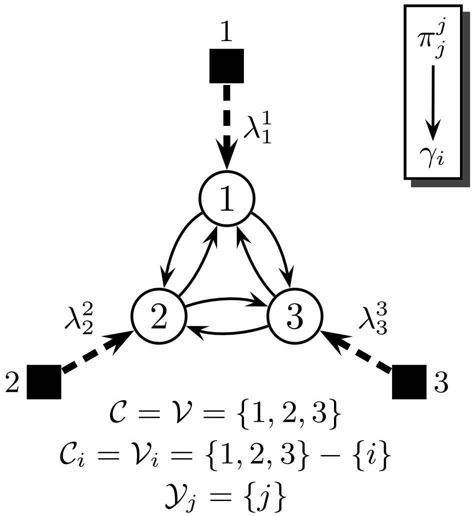


AAV patrol scenario

$\Longleftrightarrow$



AAV TPN

Metrics of volunteering

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  - $\mathcal{I}$: finite index set
  - $\Omega \triangleq \{\gamma_i\}_{i \in \mathcal{I}}$: indexed family with $\gamma_i \in [0,1]$ for each $i \in \mathcal{I}$

For $g, h \in \mathbb{N}$ and $\Gamma \subseteq \mathcal{I}$,

$$\mathrm{SOBP}_g(\Gamma) \triangleq \sum_{\ell=0}^{|\Gamma|} \frac{1}{g+\ell} \sum_{\substack{\mathcal{C} \subseteq \Gamma \\ |\mathcal{C}|=\ell}} \left(\left(\prod_{i \in \mathcal{C}} \gamma_i \right) \left(\prod_{k \in \Gamma - \mathcal{C}} (1-\gamma_k) \right) \right)$$

$$\mathrm{SOMS}_h(\Gamma) \triangleq \sum_{\ell=0}^{|\Gamma|} (-1)^{\ell} \frac{1}{h+\ell} \sum_{\substack{\mathcal{C} \subseteq \Gamma \\ |\mathcal{C}|=\ell}} \left(\prod_{i \in \mathcal{C}} \gamma_i \right)$$

Several useful relationships between SOBP and SOMS.

- To simplify presentation of combinatorial volunteering analysis, introduce SOBP and SOMS. For $\Gamma \subseteq \mathcal{A}$,

$$\begin{aligned} &\mathrm{SOBP}_1(\{i,k,\ell\} - \{i\}) \\ &= (1-\gamma_k)(1-\gamma_\ell) + \frac{1}{2}\gamma_k(1-\gamma_\ell) + \frac{1}{2}\gamma_\ell(1-\gamma_k) + \frac{1}{3}\gamma_k\gamma_\ell \end{aligned}$$

(i.e., *sum of binomial products*)

- For conveyor $j \in \mathcal{V}$ and cooperator $i \in \mathcal{C}_j = \{i,k,\ell\}$, $\mathrm{SOBP}_1(\{i,k,\ell\} - \{i\})$ is probability that i is chosen to process an advertised task from $j \in \mathcal{V}_i$ (given that it volunteered)

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- SOMS gives curvature information about SOBP
- Properties of SOMS and SOBP provide bounds for convergence analysis (i.e., Lyapunov/non-deterministic set stability)

Agent utility function – rate of gain

For $i \in \mathcal{C}$, the rate of gain

$$U_i(\gamma) \triangleq \overbrace{b_i + \underbrace{\left(1 - \prod_{j \in \mathcal{C}_i} (1 - \gamma_j)\right)}_{\Pr(\text{Volunteer from } \mathcal{C}_i \,|\, \text{Advertisement from } i)} r_i}^{\text{Conveyor part — constant with respect to } \gamma_i} + \underbrace{\gamma_i \sum_{j \in \mathcal{V}_i} \Big(\quad - \overbrace{\mathrm{SOBP}_1(\mathcal{C}_j - \{i\})}^{\Pr(i \text{ awarded task from } j \,|\, i \text{ volunteers})} c_{ij} \Big)}_{\text{Cooperator part}}$$

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where

$$b_i \triangleq \sum_{k \in \mathcal{Y}_i} \lambda_i^k \left(b_i^k - c_i^k\right)$$

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Cooperation game
Agent utility function – rate of gain

For $i \in \mathcal{C}$, the rate of gain

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Cournot oligopolies on a graph

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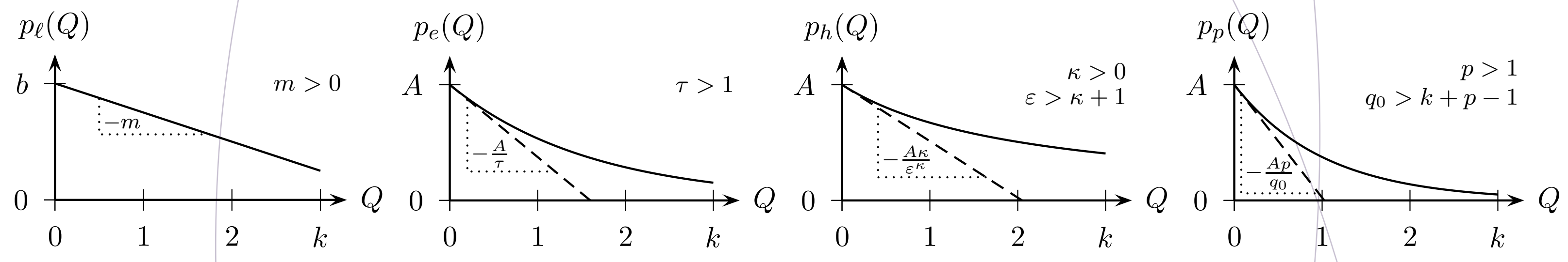
   - For $k \in \mathbb{N}$, $p''(Q) \triangleq \mathrm{d}^2 p(Q)/\mathrm{d}Q^2 > 0$ for all $Q \in [0, k]$
   - For $k \in \mathbb{N}$, $\gamma p''(Q) \leq -p'(Q)$ for all $Q \in [\gamma, k - (1 - \gamma)]$ with $\gamma \in [0, 1]$

Asynchronous convergence to Nash equilibrium
Payment and topological constraints



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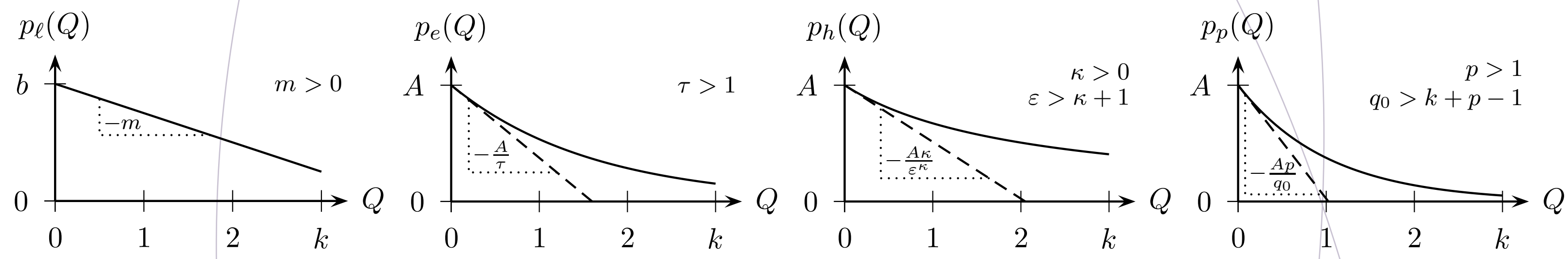

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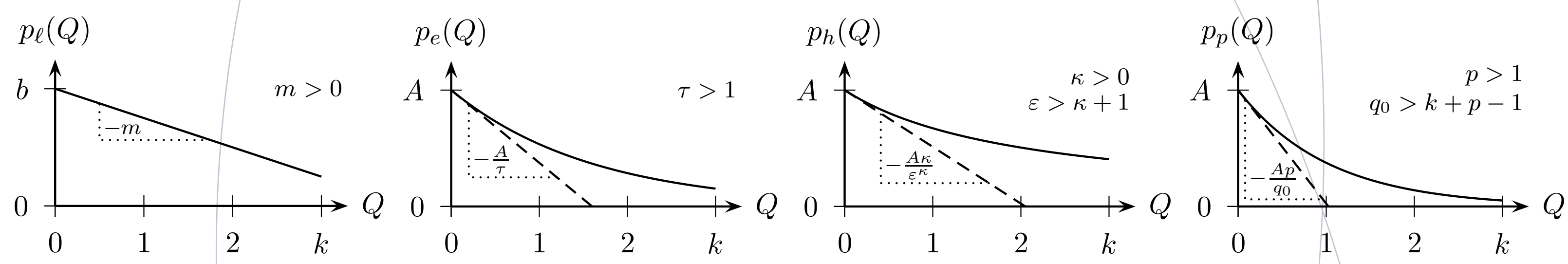

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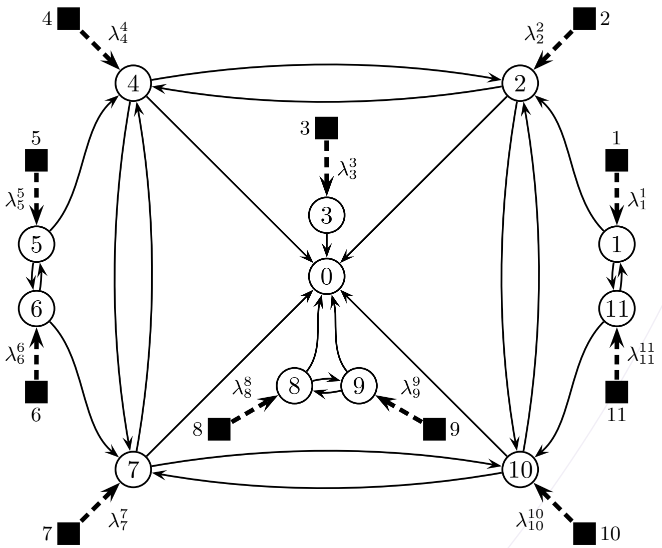
3. For cooperator $i \in \mathcal{C}$ and $j \in \mathcal{V}_i$, if j is a 3-conveyor (i.e., $|\mathcal{C}_j| = 3$), then there must be some conveyor $k \in \mathcal{V}_i$ that is a 2-conveyor

Other example stable topologies



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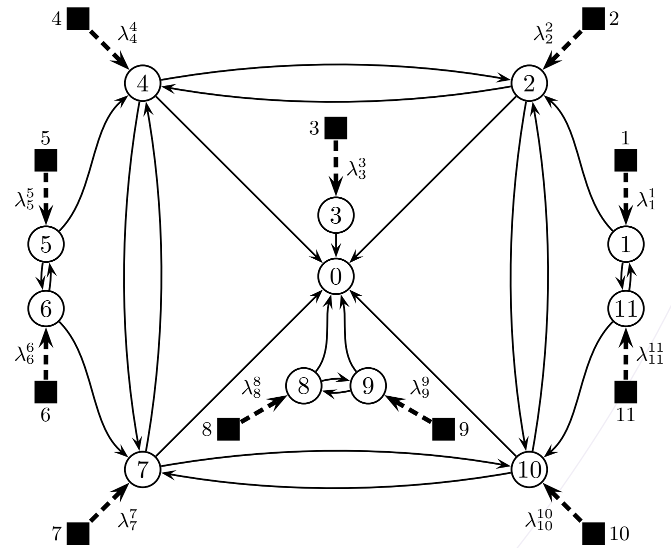

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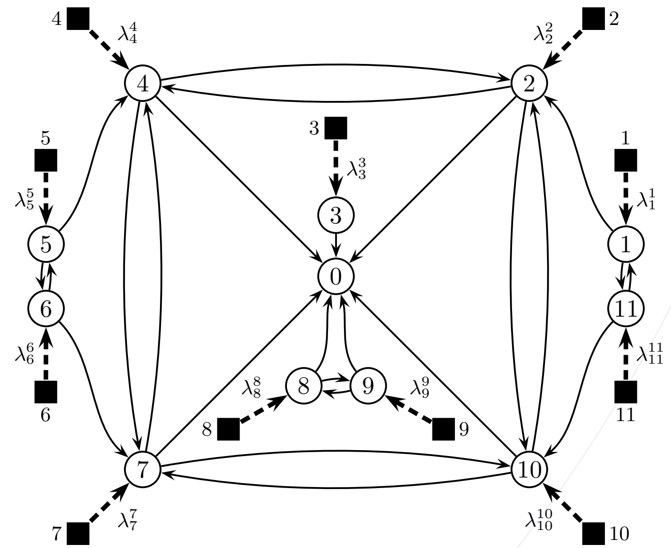

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- Future research direction: Stable network *motifs*

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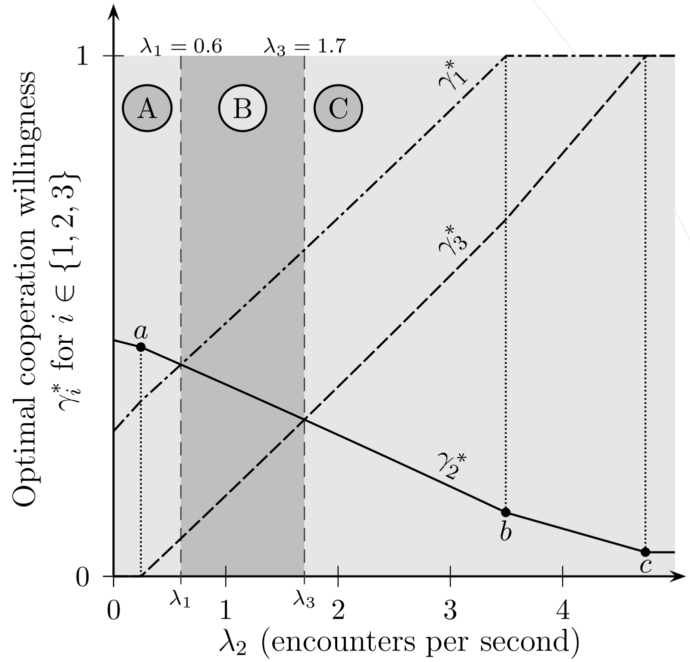
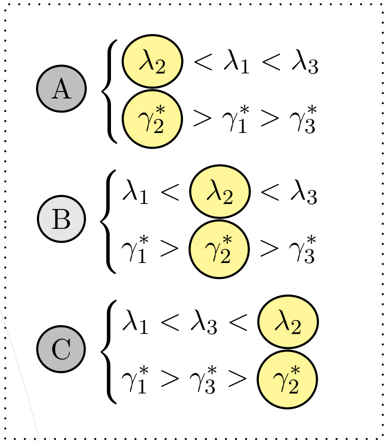
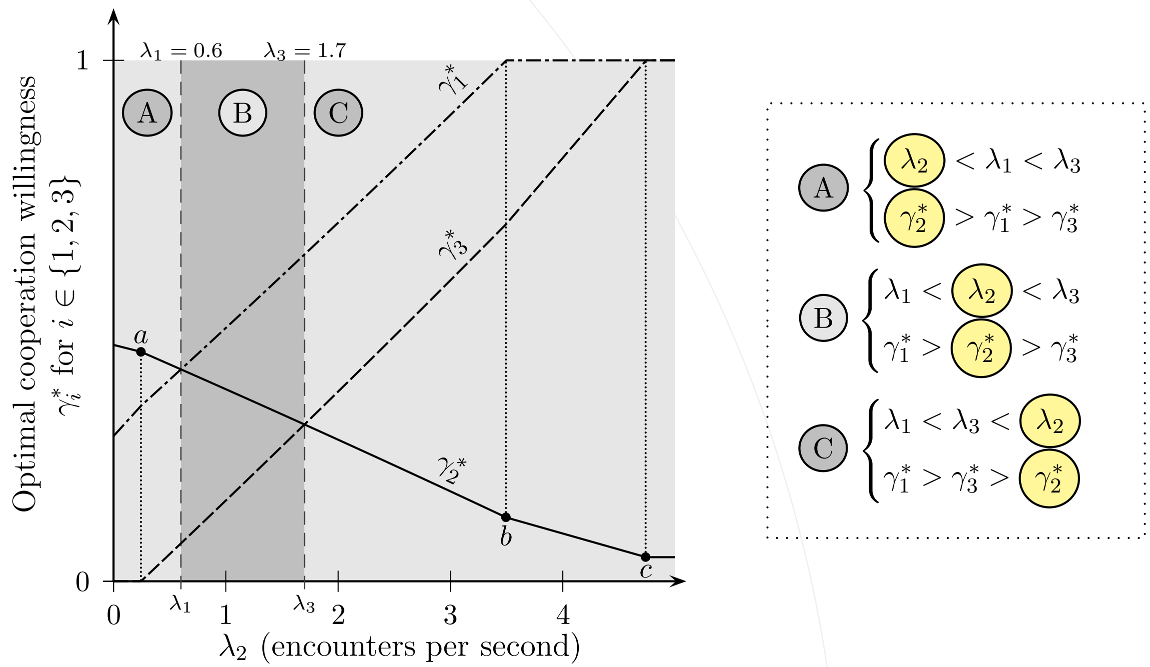




Simulation of AAV patrol scenario

- Converges to predicted Nash equilibrium

Results: cooperation by cyclic feedback



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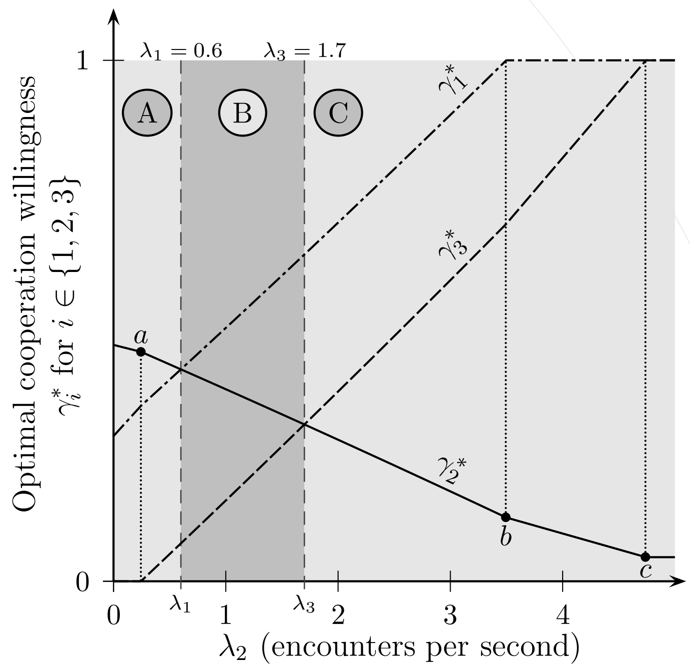

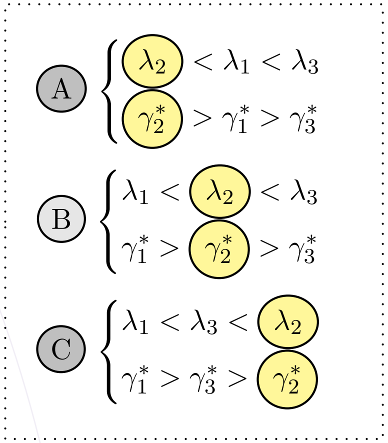

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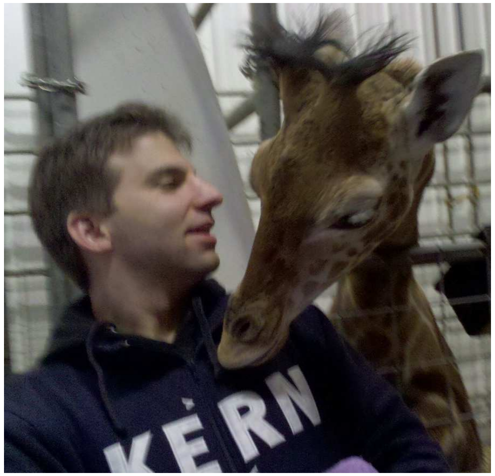
  - Provides new avenues for careers after graduate school!

Thanks!



(bringing engineers and animals together)

- Thank you!
- Helpful People: Kevin Passino, Tom Waite, Ian Hamilton
- Funding Sources:
- Questions?

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